Exercise 9

1. Let $f : \mathbb{R}^N \to \mathbb{R}$ be a strictly convex function. Suppose f has a global minimizer, show that it is unique.

2. Consider

$$\min x_1^2 + x_2^2$$
$$\text{subject to } (x_1 - 1)^2 + (x_2 - 1)^2 \leq 1, (x_1 - 1)^2 + (x_2 + 1)^2 \leq 1$$

(a) Ginve the feasible set and optimal solution x^*.
(b) Give the KKT conditions and explain whether there exists λ_1^*, λ_2^* such that $x^*, (\lambda_1^*, \lambda_2^*)$ satisfy the KKT conditions.

3. Write down the support vector machine problem. Derive the dual SVM problem and explain how to use the optimal value λ^* of dual problem to give the optimal value of the primal problem.